

**ACALANES UNION HIGH SCHOOL DISTRICT  
 COURSE OF STUDY: CURRICULAR AREA – MATH**

<u>COURSE TITLE:</u>	ALGEBRA 2/PRE-CALCULUS HONORS
<u>GRADE LEVEL:</u>	9-12
<u>COURSE LENGTH:</u>	One Year
<u>PREFERRED PREVIOUS COURSE OF STUDY:</u>	Algebra 1, Geometry Honors
<u>CREDIT:</u>	10 Credits
<u>UC/CSU CREDIT:</u>	Meets UC/CSU credit for mathematics requirement; subject area (“c”)
<u>GRADUATION REQUIREMENT:</u>	Fulfills 10 units of mathematics credit (2 semesters beyond Algebra 1) required for graduation
<u>STANDARDS AND BENCHMARKS:</u>	California State Standards Algebra 2
<u>ADOPTED:</u>	May 15, 2017
<u>INSTRUCTIONAL MATERIALS:</u>	<i>Algebra and Trig</i> , Cengage Learning

**COURSE DESCRIPTION:**

This accelerated course covers the Algebra 2 Advanced content, but in greater depth. In addition to the Algebra 2 topics that include real and complex numbers, families of functions and concepts of trigonometry, included are Pre-Calculus topics such as a complete study of conic sections, polar coordinates, vectors, parametric equations, and introductions to limits and derivatives. The course is designed to prepare students to take Calculus AB the following year. Students will be expected to solve higher order thinking problems and demonstrate procedural fluency in their responses.

**COURSE OBJECTIVES:**

**Content Objectives**

During the course the students will:

1. Extend an understanding of linear, quadratic, radical, and exponential relationships including contrasting these models
2. Develop an understanding of polynomial, logarithmic, rational, parametric, and trigonometric relationships
3. Solve polynomial, logarithmic, radical, rational, exponential, and trigonometric equations
4. Engage in methods for analyzing functions
5. Explore, manipulate, and compare different types of functions and conic sections
6. Analyze real-world data using statistics and mathematical modeling
7. Using polar coordinates to describe and perform operations with functions and complex numbers, and to graph
8. Evaluate limits of polynomial and rational functions
9. Using vectors to model and solve problems

**During the course students will work toward:**

1. Active learning through investigation and conjecture
2. Consistent study habits, organization, and personal responsibility for learning
3. Proficiency in writing about mathematics and fluency in using mathematical vocabulary
4. Strategic implementation of instructional technologies
5. Effective communication and collaboration

**ASSESSMENT:**

Assessments are designed to promote and evaluate mathematical thinking. Teachers use engaging activities that involve students in investigating, conjecturing, verifying, applying, evaluating, and communicating in various assessment modalities.

Formal and informal assessments can be made on the basis of both individual and group work. Assessments should be from a variety of means and could include performance tasks, quizzes, tests, projects, investigations, and daily assignments.

Assessments should be measuring the following claims:

*Claim #1 – Concepts & Procedures*

Students can explain and apply mathematical concepts, as well as interpret and carry out mathematical procedures with precision and fluency.

*Claim #2 – Problem Solving*

Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.

*Claim #3 – Communicating Reasoning*

Students can clearly and precisely construct viable arguments to support their own reasoning and critique the reasoning of others.

*Claim #4 – Modeling and Data Analysis*

Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

Performance Tasks

Performance tasks are used to better measure capacities such as depth of understanding, research skills, and complex analysis, which cannot be adequately assessed with selected- or constructed-response items. These tasks may require students to evaluate, optimize, design, plan, model, transform, generalize, justify, interpret, represent, estimate, and calculate solutions. Performance tasks can be used for a variety of purposes such as topic engagement, formative assessment and evaluative assessment. They may be performed individually or in small groups, depending on the purpose of the assessment.

Performance tasks should:

- Integrate knowledge and skills across multiple claims
- Require student-initiated planning and management of information and ideas
- Reflect a real-world task and/or scenario-based problem
- Allow for multiple approaches
- Represent content that is relevant and meaningful to students
- Be assessed using an understandable rubric that provides meaningful feedback for students and the teacher

GRADING GUIDELINES:

See AUHSD Grading Guidelines: Final Mark Rubric and Final Course Mark Determination Components

## COURSE CONTENT:

### Mathematical Practices

The Standards for Mathematical Practice are “habits of the mind of mathematically proficient students”. They describe the attributes that mathematics educators at all levels are striving to develop in their students, as these practices are based on key mathematical processes and proficiencies. The goal of implementing these practices is to develop students who can use their knowledge of mathematics in flexible, sophisticated, and relevant ways across multiple disciplines.

#### #1 Make sense of problems and persevere in solving them (Hypothesize & Strategize)

- Students are:
  - Making conjectures about what the problem is asking and how they can begin to solve it
  - Checking for the reasonableness of the strategy as the work progresses and making adjustments as needed
- Teachers develop this skill by having students:
  - Explain the meaning of the problem and/or restate the problem
  - Analyze the given information and develop entry points into the problem and develop strategies for solving the problem
  - Execute and evaluate multiple strategies

#### #2 Reason abstractly and quantitatively (De/Contextualize)

- Students are:
  - Determining what numbers and symbols represent through the use of diagrams, graphs or equations
- Teachers develop this skill by having students:
  - Move between multiple representations to determine the meaning behind quantities
  - Express purely mathematical expressions with real world context and taking quantities out of context and representing them as abstract mathematical ideas or expressions

#### #3 Construct viable arguments; critique others’ reasoning

- Students are:
  - Justifying their thinking by providing evidence based on mathematical properties and using that evidence to evaluate the reasoning of others
- Teachers develop this skill by having students:
  - Make conjectures, compare and contrast methods, and identify flawed logic by providing counter-example

#### #4 Model with Mathematics

- Students are:
  - Interpreting and constructing graphs, tables, number lines, diagrams or equations to model real-world situational data
- Teachers develop this skill by having students:
  - Use models to make interpolative and extrapolative inferences
  - Examine the effectiveness and appropriateness of a model

#### #5 Use appropriate tools strategically

- Students are:
  - Selecting appropriate math tools and technology to help solve problems including manipulatives, graphing utilities, tables, [matrices], computer applications, compasses, etc.
- Teachers develop this skill by having students:
  - Identify the strengths and weaknesses of different tools in relation to solving a given problem and also use tools to explore, confirm or deepen understanding

#### #6 Attend to Precision

- Students are:
  - Calculating quantities accurately through proper rounding (based on context), labeling of units of measure, and checking their work
  - Selecting a problem solving method that allows for appropriate precision
- Teachers develop this skill by having students:
  - Formulate precise explanations of their work using vocabulary and justify their rounding process
  - Re-examine their work or thinking process, and then demonstrate the method by which they check their answers

#### #7 Look For and Make Use of Structure

- Students are:
  - Looking for patterns or relationships and using that structure to simplify complex ideas
- Teachers develop this skill by having students:
  - Extend prior knowledge of similar situations to novel ones or break down complex problems in smaller parts which resemble simpler, more familiar ideas

#8 Look for and express regularity in repeated reasoning (Generalize)

- Students are:
  - Developing general methods, rules, or short cuts and determining when they are appropriate
- Teachers develop this skill by:
  - Facilitating activities which allow for students' "aha!" moments and then helping them use it to develop "rules" based on repeated trials with a process

The following table outlines all content standards and learning objectives taught in the Algebra 2 and Algebra 2 Advanced courses. Standards designated with (+) are specific for the Algebra 2 Advanced course.

Number and Quantity	
The Complex Number System (N-CN)	
Standards	Learning Objectives
<b>N-CN.1</b> <b>Perform arithmetic operations with complex numbers.</b>  1. Know there is a complex number $i$ such that $i^2 = -1$ , and every complex number has the form $a + bi$ where $a$ and $b$ real numbers.	<ul style="list-style-type: none"> <li>Understand the structure of number sets given that all real numbers are a subset of a larger group of Complex Numbers which includes imaginary numbers</li> <li>Define and use the imaginary unit <math>i</math></li> </ul>
<b>N-CN.2</b> <b>Perform arithmetic operations with complex numbers.</b>  2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	<ul style="list-style-type: none"> <li>Perform operations with complex numbers including addition, subtraction, multiplication, and division</li> <li>Use conjugates to simplify complex fractions</li> <li>Evaluate powers of <math>i</math></li> </ul>
<b>N-CN.3</b> <b>Perform arithmetic operations with complex numbers.</b>  3. Find the conjugate of a complex number	<ul style="list-style-type: none"> <li>Find the conjugate of a complex number</li> </ul>
<b>N-CN.4</b> <b>Represent complex numbers and their operations on the complex plane.</b>  4. Represent complex numbers on the complex plane in rectangular and polar form. (including real and imaginary	<ul style="list-style-type: none"> <li>Graph complex numbers on the complex plane in rectangular and polar form</li> <li>Understand how the graphs of the rectangular form and the polar form of a complex number represent the same number.</li> </ul>

<p>numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.</p>	
<p><b>N-CN.5</b>  <b>Represent complex numbers and their operations on the complex plane.</b></p> <p>5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.</p>	<ul style="list-style-type: none"> <li>• Add, subtract and multiply complex numbers geometrically</li> <li>• Show conjugates of complex numbers geometrically</li> <li>• Use DeMoivre's Theorem to find powers and roots of complex numbers and illustrate them on the complex plane in polar form</li> </ul>
<p><b>N-CN.7</b>  <b>Use complex numbers in polynomial identities and equations using polynomials with real coefficients.</b></p> <p>7. Solve quadratic equations using real coefficients that have complex solutions.</p>	<ul style="list-style-type: none"> <li>• Use the discriminant to determine if a quadratic will have complex solutions</li> <li>• Find complex solutions and zeros of quadratic equations using square roots, completing the square, and the quadratic formula</li> </ul>
<p><b>N-CN.8</b>  <b>Use complex numbers in polynomial identities and equations using polynomials with real coefficients.</b></p> <p>8. (+) Extend polynomial identities to the complex numbers. For example, rewrite <math>x^2 + 4</math> as <math>(x + 2i)(x - 2i)</math>.</p>	<ul style="list-style-type: none"> <li>• Use long/synthetic division to find all complex zeros of a polynomial given a known rational root</li> <li>• Understand the Complex Conjugate Theorem to find conjugate pairs of complex zeros</li> <li>• Write a polynomial equation as the product of complex linear factors</li> </ul>



<p><b>N-CN.9</b>  <b>Use complex numbers in polynomial identities and equations using polynomials with real coefficients.</b></p> <p>9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.</p>	<ul style="list-style-type: none"> <li>• Use Descartes' Rule of Signs to determine the possible number and nature of a polynomial's zeros</li> <li>• Use the Fundamental Theorem of Algebra to determine the number of complex solutions given the degree of a polynomial</li> </ul>
<b>Vector and Matrix Quantities (N-VM)</b>	
<p><b>N-VM.1</b>  <b>Represent and model with vector quantities</b></p> <p>1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitude.</p>	<ul style="list-style-type: none"> <li>• Describe vector quantities with direction and magnitude</li> <li>• Draw vector quantities as directed line segments</li> <li>• Use multiple forms of appropriate symbols to represent vectors and vector magnitudes</li> </ul>
<p><b>N-VM.2</b>  <b>Represent and model with vector quantities</b></p> <p>2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of the terminal point.</p>	<ul style="list-style-type: none"> <li>• Use subtraction to find a vector (in component form) between two points</li> </ul>

<p><b>N-VM.3</b>  <b>Represent and model with vector quantities</b></p> <p>3. Solve problems involving velocity and other quantities that can be represented by vectors.</p>	<ul style="list-style-type: none"> <li>• Use vectors to solve real-life problems</li> </ul>
<p><b>N-VM.4</b>  <b>Perform operations on vectors</b></p> <p>4. Add and subtract vectors</p> <ol style="list-style-type: none"> <li>Add vectors end to end, component wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.</li> <li>Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.</li> <li>Understand vector subtraction <math>\mathbf{v-w}</math> as <math>\mathbf{v+(-w)}</math> where <math>-w</math> is the additive inverse of <math>w</math>, with the same magnitude of <math>w</math> and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component wise.</li> </ol>	<ul style="list-style-type: none"> <li>• Add and subtract vectors graphically, in component form, and as the linear combination of standard unit vectors</li> <li>• Determine the magnitude and direction of the sum of two vectors</li> </ul>
<p><b>N-VM.5</b>  <b>Perform operations on vectors</b></p> <p>5. Multiply a vector by a scalar.</p> <ol style="list-style-type: none"> <li>Represent scalar multiplication graphically by scaling vectors and possibly reversing their</li> </ol>	<ul style="list-style-type: none"> <li>• Multiply a vector by a scalar graphically and in component form.</li> <li>• Determine the magnitude and direction of the product of a scalar and a vector, using both positive and negative scalars</li> </ul>

<p>direction; perform scalar multiplication component wise.</p> <p>b. Compute the magnitude <math>c\mathbf{v}</math> using <math>  c\mathbf{v}   =  c  \mathbf{v} </math>. Compute the direction of <math>c\mathbf{v}</math> knowing when <math>c\mathbf{v} \neq 0</math>, the direction of <math>c\mathbf{v}</math> of <math>c\mathbf{v}</math> is either along <math>\mathbf{v}</math> (for <math>c &gt; 0</math>) or against <math>\mathbf{v}</math> (for <math>c &lt; 0</math>).</p>	
<b>Algebra</b>	
<b>Seeing Structure in Expressions (A-SSE)</b>	
<p><b>A-SSE.1.a</b>  <b>Interpret the structure of polynomial and rational expressions.</b></p> <p>1. Interpret expressions that represent a quantity in terms of its context.</p> <p>a. Interpret parts of an expression, such as terms, factors, and coefficients</p>	<ul style="list-style-type: none"> <li>Identifying and classifying polynomials</li> <li>Define: coefficient, term, constant term, degree, standard form of a polynomial, etc.</li> </ul>
<p><b>A-SSE.1.b</b>  <b>Interpret the structure of expressions.</b></p> <p>1. Interpret expressions that represent a quantity in terms of its context.</p> <p>b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret <math>P(1 + r)^n</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</p>	<ul style="list-style-type: none"> <li>Use mathematical models to solve real-world problems</li> <li>Graph analysis and transformations</li> <li>Understand the effect of changing the value of constants within expressions</li> </ul>

<p><b>A-SSE.2</b>  <b>Interpret the structure of expressions.</b></p> <p>2. Use the structure of an expression to identify ways to rewrite it.</p>	<ul style="list-style-type: none"> <li>• Apply the order of operations</li> <li>• Solve equations for a specified variable</li> <li>• Apply the algebraic properties of equality and inequalities</li> <li>• Apply the properties of exponents</li> <li>• Employ factoring techniques for polynomials</li> <li>• Quadratics: convert standard to vertex form</li> <li>• Convert repeating decimals into rational expressions</li> </ul>
<p><b>A-SSE.4</b>  <b>Write expressions in equivalent forms to solve problems.</b></p> <p>4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems. For example, calculating mortgage payments.</p>	<ul style="list-style-type: none"> <li>• Identify common ratios and differences</li> <li>• Evaluate geometric and arithmetic series</li> <li>• Use alternate formulas to calculate the sum of geometric and arithmetic series, both finite and infinite (when possible)</li> <li>• Express and evaluate series using sigma notation</li> <li>• Use series to solve application problems</li> </ul>
<b>Arithmetic with Polynomials and Rational Expressions (A-APR)</b>	
<p><b>A-APR.1</b>  <b>Perform arithmetic operations on quadratic and higher order polynomials.</b></p> <p>1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; and add, subtract, and multiply polynomials.</p>	<ul style="list-style-type: none"> <li>• Add, subtract, and multiply polynomials</li> <li>• Divide polynomials using long and synthetic division</li> </ul>
<p><b>A-APR.2</b>  <b>Understand the relationship between zeros and factors of polynomials.</b></p> <p>2. Know and apply the Remainder Theorem: For a polynomial dividend <math>p(x)</math> and a number <math>a</math>, the remainder of</p>	<ul style="list-style-type: none"> <li>• Apply the remainder and factor theorems to determine the zeros of a polynomial</li> <li>• Analyze and use depressed polynomials to factor polynomials completely</li> <li>• Use the Remainder Theorem combined with synthetic division to find polynomial function values - <math>p(a)</math></li> </ul>

the division by divisor $(x - a)$ is $p(a)$ . Thus $p(a) = 0$ if and only if the divisor $(x - a)$ is a factor of $p(x)$ .	
<b>A-APR.3</b> <b>Understand the relationship between zeros and factors of polynomials.</b>  3. Identify the zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	<ul style="list-style-type: none"> <li>• Understand and apply the Fundamental theorem of Algebra</li> <li>• Find roots and zeros of polynomials</li> <li>• Interpret and then graph polynomial functions</li> <li>• Understand end behavior of polynomial functions</li> </ul>
<b>A-APR.4</b> <b>Use polynomial identities to solve problems.</b>  4. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.	<ul style="list-style-type: none"> <li>• Solve polynomial equations</li> <li>• Find rational roots</li> <li>• Find complex roots (interpret discriminants)</li> <li>• Use the complex conjugates theorem</li> <li>• Understand and apply the rational root theorem to list and test possible zeros</li> <li>• Use Descartes' rule of signs</li> </ul>
<b>A-APR.5</b> <b>Use polynomial identities to solve problems.</b>  5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of $x$ and $y$ for a positive integer $n$ , where $x$ and $y$ are any numbers with coefficients determined for example by Pascal's triangle.	<ul style="list-style-type: none"> <li>• Use Pascal's triangle to expand binomials</li> <li>• Use the binomial expansion theorem</li> <li>• Relate Pascal's triangle to the binomial expansion theorem</li> <li>• Use the binomial expansion theorem to find individual terms of a polynomial</li> </ul>
<b>A-APR.6</b> <b>Rewrite rational expressions including but not limited to linear and quadratic denominators.</b>  6. Rewrite simple rational expressions in different forms. For example, write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$ , where $a(x)$ , $b(x)$ , $q(x)$ , and $r(x)$ are polynomials with the	<ul style="list-style-type: none"> <li>• Simplify rational expressions</li> <li>• Divide polynomials using long and synthetic division</li> <li>• Use the remainder theorem to write polynomials as a product of the quotient and divisor, plus the remainder</li> <li>• Add, subtract, multiply, and divide rational expressions</li> <li>• Rationalize the denominator of rational expressions</li> </ul>

degree of $r(x)$ less than the degree of $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.	
<b>A-APR.7</b> <b>Rewrite rational expressions including but not limited to linear and quadratic denominators.</b>  7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	<ul style="list-style-type: none"> <li>• + State the excluded values for simplified rational expressions</li> <li>• + State the excluded values for simplified rational expressions, distinguishing between point and infinite discontinuity</li> </ul>
<b>Creating Equations (A-CED)</b>	
<b>A-CED.1</b> <b>Create equations, including but not limited to simple root functions, that describe numbers or relationships.</b>  1. Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	<ul style="list-style-type: none"> <li>• Write equations and inequalities of linear, quadratic, absolute value, rational, radical, and exponential functions in a single variable.</li> <li>• Solve real-world problems in a single variable.</li> </ul>
<b>A-CED.2</b> <b>Create equations, including but not limited to simple root functions, that describe numbers or relationships.</b>  2. Create equations in two or more variables to represent relationships between quantities, and graph those equations on coordinate axes with labels and scales.	<ul style="list-style-type: none"> <li>• Write equations and inequalities of linear, quadratic, absolute value, rational, radical, and exponential functions in two or more variables.</li> <li>• Solve real-world problems in two or more variables.</li> </ul>

<p><b>A-CED.3</b>  <b>Create equations, including but not limited to simple root functions, that describe numbers or relationships.</b></p> <p>3. Represent constraints by equations or inequalities, and by systems of equations or inequalities, and interpret the solutions as viable or nonviable options in a modeling context.</p>	<ul style="list-style-type: none"> <li>• Write and solve systems of equations in two and three variables</li> <li>• Interpret solutions within context and apply units appropriately</li> <li>• Linear Programming</li> </ul>
<p><b>A-CED.4</b>  <b>Create equations, including but not limited to simple root functions, that describe numbers or relationships.</b></p> <p>4. Rearrange formulas to highlight a quantity of interest using the same reasoning for solving equations.</p>	<ul style="list-style-type: none"> <li>• Solve systems using the substitution method (?)</li> <li>• Find the inverse of a function</li> </ul>
<b>Reasoning with Equations and Inequalities (A-REI)</b>	
<p><b>A-REI.2</b>  <b>Understand and explain solving simple, radical, and rational equations as a process of reasoning.</b></p> <p>2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.</p>	<ul style="list-style-type: none"> <li>• Solve radical equations (including rational exponents) and inequalities</li> <li>• Identify extraneous solutions</li> <li>• Solve rational equations by cross multiplying and by using the least common denominator</li> <li>• State the domain of radical equations</li> </ul>
<p><b>A-REI.3.1</b>  <b>Solve equations and inequalities in one variable.</b></p> <p>3.1 Solve, graph, and interpret one-variable, absolute value equations and inequalities.</p>	<ul style="list-style-type: none"> <li>• Solve one-variable equations and inequalities involving absolute value, graphing the solutions and interpreting them in context.</li> </ul>

<p><b>A-REI.8</b>  <b>Solve systems of equations</b></p> <p>8. Represent a system of linear equations as a single matrix equation in a vector variable</p>	<ul style="list-style-type: none"> <li>● Rewrite a system of linear equations in matrix form</li> </ul>
<p><b>A-REI.9</b>  <b>Solve systems of equations</b></p> <p>9. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (use technology for matrices 3x3 or greater)</p>	<ul style="list-style-type: none"> <li>● Find the inverse of a 2x2 matrix (if it exists) without technology</li> <li>● Find the inverse of a 3x3 matrix (if it exists) using technology</li> <li>● Use the system of matrices in matrix form and the inverse to solve systems of linear equations</li> </ul>
<p><b>A-REI.11</b>  <b>Represent and solve equations and inequalities graphically, combining polynomial, rational, radical, absolute value, and exponential functions.</b></p> <p>11. Explain why the x-coordinates of the points where the graphs of the equations <math>y = f(x)</math> and <math>y = g(x)</math> intersect are the solutions of the equation <math>f(x) = g(x)</math>; find approximate solutions (e.g., using technology to graph the functions, make tables of values, or find successive approximations); include cases where functions are linear, polynomial, rational, absolute value, exponential, and logarithmic.</p>	<ul style="list-style-type: none"> <li>● Solve systems that combine linear, polynomial, rational, radical, absolute value, exponential, and logarithmic functions by creating tables and using technology to graph functions and find approximate solutions.</li> <li>● Interpret the point(s) of intersection for two or more equations as the solution to the system with and without the use of technology</li> <li>● Solve rational inequalities</li> </ul>



Functions	
Interpreting Functions (F-IF)	
<p><b>F-IF.4</b>  <b>Interpret functions that arise in applications in terms of the context, emphasizing the selection of appropriate models.</b></p> <p>4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</p>	<ul style="list-style-type: none"> <li>• Solve systems by graphic and algebraic methods.</li> <li>• Analyze graphs of quadratics with focus and directrix</li> <li>• Graph polynomial and rational functions</li> <li>• Analyze the graphs of exponential functions, including using transformations</li> <li>• Solve exponential equations</li> <li>• Graph <math>y = \sin x</math>, <math>y = \cos x</math>, and <math>y = \tan x</math>; analyze key characteristics</li> <li>• Apply transformations (amplitude, Vertical Shift, Periodic Functions, Period Change, Phase Shift) to trigonometric functions</li> <li>• Find maximum and minimum values of quadratic functions</li> <li>• Identify intervals for which the function is increasing/decreasing</li> <li>• Graph quadratic and polynomial functions using x-intercepts</li> <li>• Describe the end behavior of functions</li> <li>• Write equations of parabolas</li> <li>• Identify polynomial functions</li> <li>• Identify even and odd functions</li> <li>• Identify the periodicity of periodic functions</li> </ul>
<p><b>F-IF.5</b>  <b>Interpret functions that arise in applications in terms of the context emphasizing the selection of appropriate models.</b></p> <p>5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.</p>	<ul style="list-style-type: none"> <li>• Graph polynomial, rational and exponential and trigonometric functions and analyze key characteristics, including, but not limited to domain and range</li> </ul>

<p><b>F-IF.6</b>  <b>Interpret functions that arise in applications in terms of the context emphasizing the selection of appropriate models.</b></p> <p>6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</p>	<ul style="list-style-type: none"> <li>• Graph linear, quadratic and exponential equations and determine the rate of change between specified points</li> <li>• Use data to create a mathematical model (such as regression)</li> </ul>
<p><b>F-IF.7.b</b>  <b>Analyze functions using different representations focusing on using key features to guide selection of appropriate model function types.</b></p> <p>7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.  b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</p>	<ul style="list-style-type: none"> <li>• Identify the key characteristics and graph radical, piecewise, step, and absolute value functions</li> </ul>
<p><b>F-IF.7.c</b>  <b>Analyze functions using different representations focusing on using key features to guide selection of appropriate model function types.</b></p> <p>7. Graph functions expressed symbolically and show key features of the graph (by hand in simple cases and using technology for more complicated cases.)  c. Graph polynomial functions, identifying the zeros when suitable factorizations are available, and show end behavior.</p>	<ul style="list-style-type: none"> <li>• Graph polynomial functions (including quadratic)</li> <li>• Identify the key characteristic of graphs including end behavior and zeros</li> </ul>

<p><b>F-IF.7.d</b>  <b>Analyze functions using different representations focusing on using key features to guide selection of appropriate model function types.</b></p> <p>7. Graph functions expressed symbolically and show key features of the graph (by hand in simple cases and using technology for more complicated cases.)  d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and show the end behavior.</p>	<ul style="list-style-type: none"> <li>● Identify zeros, intercepts, as well as vertical, horizontal and slant asymptotes of a rational function</li> <li>● Use the key features that have been identified to graph a rational function, and show the end behavior</li> <li>● Identify the key features on the graph of a rational function.</li> <li>● Use limits to determine the horizontal asymptotes of the graphs of rational functions</li> </ul>
<p><b>F-IF.7.e</b>  <b>Analyze functions using different representations focusing on using key features to guide selection of appropriate model function types.</b></p> <p>7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.  e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</p>	<ul style="list-style-type: none"> <li>● Graph exponential growth and decay functions</li> <li>● Graph natural base functions</li> <li>● Graph logarithmic functions</li> <li>● Graph and explore characteristics of the six primary trigonometric functions.</li> <li>● Graph inverse trigonometric functions</li> <li>● Graph transformations of trigonometric functions, showing period, amplitude, the sinusoidal axis and phase shift</li> </ul>
<p><b>F-IF.8</b>  <b>Analyze functions using different representations focusing on using key features to guide selection of appropriate model function types.</b></p>	<ul style="list-style-type: none"> <li>● Rewrite functions into different but equivalent forms, e.g. vertex versus standard form for quadratics</li> <li>● Identify the key characteristics of the graph by analyzing the equation</li> <li>● Given a sinusoidal graph, write the equation in multiple forms, including sine or cosine with negative coefficient or phase shift</li> </ul>

<p>8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function</p>	
<p><b>F-IF.9</b>  <b>Analyze functions using different representations focusing on using key features to guide selection of appropriate model function types.</b></p> <p>9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).</p>	<ul style="list-style-type: none"> <li>• Compare two functions that are in different representations, noting key differences in their characteristics. For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</li> </ul>
<p><b>F-IF.10</b>  <b>Analyze functions using different representations focusing on using key features to guide selection of appropriate model function types.</b></p> <p>10. (+) Demonstrate an understanding of functions and equations defined parametrically and graph them</p>	<ul style="list-style-type: none"> <li>• (+) Demonstrate an understanding of functions and equations defined parametrically and graph them by building a table of values, and by eliminating the parameter and adjusting the domain</li> </ul>
<p><b>F-IF.11</b>  <b>Analyze functions using different representations focusing on using key features to guide selection of appropriate model function types.</b></p> <p>11. (+) Graph polar coordinates and curves. Convert between polar and rectangular coordinate systems.</p>	<ul style="list-style-type: none"> <li>• (+) Graph polar coordinates and curves. Convert between polar and rectangular coordinate systems.</li> </ul>

<b>Building Functions (F-BF)</b>	
<p><b>F-BF.1.B</b>  <b>Build a function that models the relationship between two quantities or functions.</b></p> <p>1. Write a function that describes a relationship between two quantities.  b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</p>	<ul style="list-style-type: none"> <li>Write a function which models a relationship between two variables (e.g. model for projectile motion)</li> </ul>
<p><b>F-BF.1.c</b>  <b>Build a function that models the relationship between two quantities or functions.</b></p> <p>1. Write a function that describes a relationship between two quantities.  c. Compose functions. <i>For example, if <math>T(y)</math> is the temperature in the atmosphere as a function of height, and <math>h(t)</math> is the height of a weather balloon as a function of time, then <math>T(h(t))</math> is the temperature at the location of the weather balloon as a function of time.</i></p>	<ul style="list-style-type: none"> <li>Write the composition of two functions and use in application problems</li> </ul>
<p><b>F-BF.3</b>  <b>Build new functions from existing functions, including but not limited to simple, radical, rational, and exponential functions, and emphasize the common effect of each transformation across function types.</b></p>	<ul style="list-style-type: none"> <li>Identify transformations of a function from a graph</li> <li>Define the effect of a transformation on the graph of a function including amplitude and vertical shifts.</li> <li>Explore the effect of constants on the graph of an equation through the use of technology.</li> <li>Prove algebraically that a function is even or odd.</li> </ul>

<p>3. Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>kf(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</p>	<ul style="list-style-type: none"> <li>Graphically identify if a function is even or odd.</li> <li>Define the effect of a transformation on the graph of a function including amplitude, vertical shifts, and phase shifts</li> <li>Algebraically and graphically identify sine and cosine as even and odd functions</li> </ul>
<p><b>F-BF.4</b>  <b>Build new functions from existing functions, including but not limited to simple, radical, rational, and exponential functions, and emphasize the common effect of each transformation across function types.</b></p> <p>4. Find inverse functions.  a. Solve an equation of the form <math>f(x) = c</math> for a simple function <math>f</math> that has an inverse and write an expression for the inverse. For example, <math>f(x) = 2x^3</math> or <math>f(x) = (x + 1)/(x - 1)</math> for <math>x \neq 1</math>.  b. (+) Verify by composition that one function is the inverse of another.</p>	<ul style="list-style-type: none"> <li>Find the inverse of a function.</li> <li>Verify the inverses of nonlinear functions using composition of functions</li> <li>Use inverse properties of logarithmic and exponential functions to solve equations</li> <li>Define and evaluate inverse trigonometric functions based on restricted domains</li> <li>Use inverse trigonometric functions to solve trigonometric equations</li> </ul>
<p><b>F-BF.5</b>  Build new functions from existing functions, including but not limited to simple, radical, rational, and exponential functions, and emphasize the common effect of each transformation across function types.</p> <p>(+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.</p>	<ul style="list-style-type: none"> <li>Understand the inverse relationships between exponential and logarithmic functions and use it to rewrite logarithmic and exponential expressions in the inverse functions form in order to evaluate and to solve equations.</li> </ul>

<b>Linear, Quadratic, and Exponential Models (F-LE)</b>	
<b>F-LE.4</b> <b>Construct and compare linear, quadratic, and exponential models and solve problems.</b>  4. For exponential models, logarithmically express the solution to $ab^{ct} = d$ where $a$ , $c$ , and $d$ are numbers and the base $b$ is 2, 10, or $e$ . In addition, use technology to evaluate logarithms as solutions for exponentials.	<ul style="list-style-type: none"> <li>• Convert between exponential and logarithmic form</li> <li>• Evaluate exponential and logarithmic expressions using technology</li> <li>• Solve growth and decay applications problems</li> </ul>
<b>F-LE.4.1</b> 4.1 Prove simple laws of logarithms.	<ul style="list-style-type: none"> <li>• Prove simple laws of logarithms</li> </ul>
<b>F-LE.4.2</b> 4.2 Use the definition of logarithms to translate between logarithms in any base.	<ul style="list-style-type: none"> <li>• Apply the change of base formula to evaluate logarithmic expressions</li> </ul>
<b>F-LE.4.3</b> 4.3 Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.	<ul style="list-style-type: none"> <li>• Understand the relationship between the laws of exponents and the laws of logarithms</li> <li>• Expand, condense, and evaluate logarithms using and applying the laws of logarithms and exponents</li> </ul>
<b>Trigonometric Functions (T-TF)</b>	
<b>F-TF.1</b> <b>Extend the domain of trigonometric functions using the unit circle.</b>  1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	<ul style="list-style-type: none"> <li>• Define and draw angles in standard position</li> <li>• Define radian measure</li> <li>• Convert between degree and radian measure</li> </ul>

<p><b>F-TF.2</b>  <b>Extend the domain of trigonometric functions using the unit circle.</b></p> <p>2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.</p>	<ul style="list-style-type: none"> <li>• Understand that coterminal angles define the domain of trigonometric function as <math>(-\infty, \infty)</math></li> <li>• Define positive, negative, clockwise, and counterclockwise angles.</li> <li>• Identify and use reference angles to evaluate trigonometric functions</li> <li>• Utilize special right triangles in the unit circle to evaluate trigonometric functions of any angle</li> </ul>
<p><b>F-TF.3</b>  <b>Extend the domain of trigonometric functions using the unit circle.</b></p> <p>(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for <math>\pi/3</math>, <math>\pi/4</math> and <math>\pi/6</math>, and use the unit circle to express the values of sine, cosine, and tangent for <math>x</math>, <math>\pi + x</math>, and <math>2\pi - x</math> in terms of their values for <math>x</math>, where <math>x</math> is any real number.</p>	<ul style="list-style-type: none"> <li>• Understand that the sine, cosine and tangent of any radian multiple of <math>\pi/3</math>, <math>\pi/4</math> and <math>\pi/6</math> can be found by identifying the reference angle as <math>\pi/3</math>, <math>\pi/4</math> or <math>\pi/6</math> and using the ratios of special right triangles</li> </ul>
<p><b>F-TF.3</b>  <b>Extend the domain of trigonometric functions using the unit circle.</b></p> <p>(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.</p>	<ul style="list-style-type: none"> <li>• Understand the quadrants in which sine, cosine and tangent are positive and negative, and use this knowledge to determine whether each function is even or odd.</li> </ul>
<p><b>F-TF.2.1</b>  2.1 Graph the six primary trigonometric functions.</p>	<ul style="list-style-type: none"> <li>• Identify the characteristics of parent functions <math>y = \sin x</math>, <math>y = \cos x</math> and <math>y = \tan x</math></li> <li>• Use transformations to change the period and amplitude</li> <li>• Use transformations to translate parent functions</li> <li>• Use parent functions and transformations to graph <math>y = \sec x</math>, <math>y = \csc x</math> and <math>y = \cot x</math></li> <li>• Use transformations to change the period and amplitude and phase shift</li> <li>• Understand the relationship between phase shift and <math>y = \sin x</math> and <math>y = \cos x</math></li> </ul>



<p><b>F-TF.5</b>  <b>Model periodic phenomena with trigonometric functions.</b></p> <p>5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.</p>	<ul style="list-style-type: none"> <li>• Understand the role of <math>a, b</math> and <math>d</math> in the equation <math>y = a \sin(bx) + d</math> to determine the frequency, amplitude, vertical shifts of sinusoidal models</li> <li>• Include phase shift in the understanding of <math>a, b, c</math> and <math>d</math> in the graph of <math>y = a \sin b(x + c) + d</math></li> <li>• Write and use trigonometric equations for sinusoidal modeling problems</li> </ul>
<p><b>F-TF.6</b>  <b>Model periodic phenomena with trigonometric functions.</b></p> <p>6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.</p>	<ul style="list-style-type: none"> <li>• Understand that inverse trigonometric functions are not functions and define the restricted domain of <math>\arcsin</math>, <math>\arccos</math> and <math>\arctan</math> to make them functions</li> </ul>
<p><b>F-TF.6</b>  <b>Model periodic phenomena with trigonometric functions.</b></p> <p>(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*</p>	<ul style="list-style-type: none"> <li>• Use the principle values of the <math>\arcsin</math>, <math>\arccos</math> and <math>\arctan</math> in the restricted domain to determine all solutions on the interval <math>[0, 2\pi]</math>.</li> </ul>
<p><b>F-TF.8</b>  <b>Prove and apply trigonometric identities.</b></p> <p>8. Prove the Pythagorean identity <math>\sin^2(\theta) + \cos^2(\theta) = 1</math> and use it to find <math>\sin(\theta)</math>, <math>\cos(\theta)</math>, or <math>\tan(\theta)</math> given <math>\sin(\theta)</math>, <math>\cos(\theta)</math>, or <math>\tan(\theta)</math> and the quadrant of the angle.</p>	<ul style="list-style-type: none"> <li>• Use <math>x = \cos(\theta)</math> and <math>y = \sin(\theta)</math> from the unit circle to derive the Pythagorean identity</li> <li>• Use trigonometric identities to evaluate trigonometric functions, simplify trigonometric expressions, and verify other trigonometric identities</li> <li>• Use the sum and difference, and double and half-angle identities to evaluate trigonometric functions, simplify trigonometric expressions, and verify other trigonometric identities</li> <li>• Apply trigonometric identities to solve trigonometric equations</li> </ul>

Geometry	
Expressing Geometric Properties with Equations (G-GPE)	
<p><b>G-GPE.3.1</b>  <b>Translate between the geometric description and the equation for a conic section.</b></p> <p>3.1 Given a quadratic equation of the form <math>Ax^2 + By^2 + Cx + Dy + E = 0</math>, use the completing the square method to put the equation into standard form; identify whether the graph of the equation is a circle, ellipse, parabola, or hyperbola; and graph the equation. In Algebra II, this standard addresses only circles and parabolas.</p>	<ul style="list-style-type: none"> <li>• Use the completing the square method, rewrite equations from general to standard form for circles and parabolas to identify the center, vertex, and radius, as appropriate.</li> <li>• Identify a conic section as a circle, ellipse, parabola or hyperbola by examining an equation in general form</li> <li>• Graph conic sections and illustrate the center, vertex, focus, directrix, and asymptotes, as appropriate</li> <li>• Find the equations of the asymptotes of a hyperbola</li> </ul>
Statistics and Probability	
Interpreting Categorical and Quantitative Data (S-ID)	
<p><b>S-ID.4</b>  <b>Summarize, represent, and interpret data on a single count or measurement variable.</b></p> <p>4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.</p>	<ul style="list-style-type: none"> <li>• Calculate probabilities using normal distributions</li> <li>• Use z-scores and the standard normal table to find probabilities</li> <li>• Recognize normal data sets</li> </ul>

<b>Making Inferences and Justifying Conclusions (S-IC)</b>	
<p><b>S-IC.1</b>  <b>Understand and evaluate random processes underlying statistical experiments.</b></p> <p>1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.</p>	<ul style="list-style-type: none"> <li>• Distinguish between populations and samples</li> <li>• Analyze hypotheses</li> </ul>
<p><b>S-IC.2</b>  <b>Understand and evaluate random processes underlying statistical experiments.</b></p> <p>2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, if a model says a spinning coin falls heads up with probability 0.5, would a result of five tails in a row cause one to question the model?</p>	<ul style="list-style-type: none"> <li>• Analyze types of sampling methods</li> <li>• Resample data using a simulation to analyze a hypothesis</li> </ul>
<p><b>S-IC.3</b>  <b>Make inferences and justify conclusions from sample surveys, experiments, and observational studies.</b></p> <p>3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.</p>	<ul style="list-style-type: none"> <li>• Analyze methods of collecting data</li> <li>• Recognize bias in survey questions and sampling</li> <li>• Recognize how randomization applies to experiments and observational studies</li> </ul>

<b>S-IC.4</b> <b>Make inferences and justify conclusions from sample surveys, experiments, and observational studies.</b>  4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	<ul style="list-style-type: none"> <li>Find margins of error for surveys</li> <li>Estimate and analyze population parameters</li> </ul>
<b>S-IC.5</b> <b>Make inferences and justify conclusions from sample surveys, experiments, and observational studies.</b>  5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	<ul style="list-style-type: none"> <li>Organize data from an experiment with two samples</li> <li>Make inferences about a treatment</li> </ul>
<b>S-IC.6</b> <b>Make inferences and justify conclusions from sample surveys, experiments, and observational studies.</b>  6. Evaluate reports based on data	<ul style="list-style-type: none"> <li>Evaluate reports based on data</li> </ul>
<b>Using Probability to Make Decisions (S-MD)</b>	
<b>S-MD.6+</b> <b>Use probability to evaluate simple and complex outcomes of decisions.</b>  6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	<ul style="list-style-type: none"> <li>Prove whether events are independent or dependent using the sample space</li> <li>Calculate probabilities of independent and dependent events</li> <li>Calculate conditional probabilities</li> <li>Find probabilities of compound events</li> <li>Use more than one probability rule to solve real-life problems</li> </ul>

<p><b>S-MD.7+</b>  <b>Use probability to evaluate simple and complex outcomes of decisions.</b></p> <p>7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).</p>	<ul style="list-style-type: none"> <li>• Use more than one probability rule to solve real-life problems</li> <li>• Construct and interpret probability and binomial distributions</li> </ul>
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